

VECTOR FORM OF A MULTIDIMENSIONAL MATHEMATICAL MODEL FOR MONITORING AND PREDICTING THE IN-SITU LEACHING PROCESS IN A POROUS MEDIUM.

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Abstract

This paper analyzes the hydrodynamic processes involved in in-situ leaching, specifically the acid-based extraction of precious metals from ore deposits. A mathematical model, based on filtration-convection, and diffusion processes inherent in underground leaching, has been developed to comprehensively study, monitor, and forecast object behavior. This model incorporates the impact of varying hydrodynamic parameters, specifically the filtration coefficient and medium porosity, which are functions of pressure levels and process kinetics. The primary research objective is groundwater protection from pollution sources.

Keywords: *In-situ leaching, mathematical modeling, minerals, filtration, diffusion, kinetics, useful component.*

Introduction

The discovery, development, and efficient utilization of valuable geological resources remain a critical priority for all societies. The technological advancements of our era necessitate the extraction and application of subsurface mineral reserves. Consequently, research into efficient mineral extraction methodologies is paramount. The adoption of in-situ leaching (ISL) as an advanced technique is gaining momentum, thus demanding sophisticated mathematical modeling of the subsurface processes involved. This necessitates the development of increasingly complex mathematical models (MM) requiring advanced computational resources for numerical solutions.

Taking into account the above, this paper proposes a mathematical model to solve pressure and kinetic-dependent problems within an ore deposit.

Physical statement of the problem

Based on the above, to study the underground leaching process, it is necessary to determine the concentration function of the useful component of the $C_2(x, y, z, t)$ in a limited heterogeneous region

$$G = \{(x, y, z, t), 0 < x < L_x, 0 < y < L_y, 0 < z < L_z, 0 < t \leq T\}.$$

In this case, the propagation of the pressure field $H(x, y, z, t)$ is determined from the equations of the elastic filtration regime:

$$\begin{aligned} \operatorname{div}(k(x, y, z)h(x, y, z)W) &= \beta m(x, y, z)h(x, y, z)\frac{\partial H(x, y, z, t)}{\partial t} + f(x, y, z, t), \\ \operatorname{grad}(H) &= W, \quad (x, y, z) \in G. \end{aligned} \quad (1)$$

The filtration rates are determined by Darcy's law

$$\overline{V(x, y, z, t)} = (V_x(x, y, z, t), V_y(x, y, z, t), V_z(x, y, z, t)) = -k(x, y, z)\operatorname{grad}(H(x, y, z, t)). \quad (2)$$

The propagation of the reactant field is determined by solving the convective diffusion equation

$$\begin{aligned} &\frac{\partial}{\partial x} \left[D_{xx}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial x} + D_{xy}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial y} + D_{xz}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial z} \right] + \\ &+ \frac{\partial}{\partial y} \left[D_{yy}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial y} + D_{yx}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial x} + D_{yz}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial z} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_{zz}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial z} + D_{zx}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial x} + D_{zy}(x, y, z) \frac{\partial C_1(x, y, z, t)}{\partial y} \right] - \\ &- \frac{\partial(V_x(x, y, z, t)C_1(x, y, z, t))}{\partial x} - \frac{\partial(V_y(x, y, z, t)C_1(x, y, z, t))}{\partial y} - \frac{\partial(V_z(x, y, z, t)C_1(x, y, z, t))}{\partial z} = \\ &= m_g \frac{\partial C_1(x, y, z, t)}{\partial t}, \quad [x, y, z] \in G \end{aligned} \quad (3)$$

The desired distribution of the concentration function of the useful component is determined by solving the following equation

$$\begin{aligned} &\frac{\partial}{\partial x} \left[D_{xx}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial x} + D_{xy}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial y} + D_{xz}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial z} \right] + \\ &+ \frac{\partial}{\partial y} \left[D_{yy}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial y} + D_{yx}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial x} + D_{yz}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial z} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_{zz}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial z} + D_{zx}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial x} + D_{zy}(x, y, z) \frac{\partial C_2(x, y, z, t)}{\partial y} \right] - \\ &- \frac{\partial(V_x(x, y, z, t)C_2(x, y, z, t))}{\partial x} - \frac{\partial(V_y(x, y, z, t)C_2(x, y, z, t))}{\partial y} - \frac{\partial(V_z(x, y, z, t)C_2(x, y, z, t))}{\partial z} = \\ &= m_c \frac{\partial C_2(x, y, z, t)}{\partial t} + \frac{\partial N}{\partial t}, \quad [x, y, z] \in G \end{aligned} \quad (4)$$

The equation of mass exchange kinetics, which determines the rate of transition of a substance from one phase to another, has the following form:

$$\begin{aligned} \frac{\partial N(x, y, t)}{\partial t} &= \gamma(C_1)f(C_2, N, t), \\ N(x, y, t) &= N^0(x, y), \quad t = 0, \quad [x, y] \in G. \end{aligned} \quad (13)$$

Analysis of the problem statement reveals that variations in pressure within the ore deposit, induced by solution injection and extraction, directly impact the permeability and porosity coefficients of the stratum.

The rate of change in these hydrodynamic parameters is pressure-dependent; exhibiting exponential behavior under high pressure and linear behavior under low pressure conditions.

It should be noted that during the underground leaching process, due to the effect of the reagent on the ore deposits, a chemical reaction occurs and the substance passes from one phase to another, as a result of which the hydrodynamic parameters of the porous medium (filtration and porosity coefficients) and the pressure in the ore collector change.

As noted in [7], the change in porosity depending on the pressure can be expressed using the equation:

$$m = m_0 + \beta_c (H - H_0),$$

and with significant changes in pressure, the change in porosity is described by the equation

$$m = m_0 e^{-\beta_c (H_0 - H)/m_0},$$

where m_0 is porosity coefficient at $H = H_0$.

The analysis of the experimental data showed that not only the porosity, but also the permeability of the ore reservoir significantly depends on the formation pressure and, with small changes in its value, this can be written as a linear dependence:

$$k = k_0 [1 - a_K (H_0 - H)],$$

and for large ones - exponential

$$k = k_0 e^{-a_K (H - H_0)},$$

where: $H(x, y, z, t)$ is pressure value, (m); $H^0(x, y, z)$ is initial pressure value, (m); $\phi(x, y, z, t)$ value of pressure at the boundary, (m). m is porosity coefficient value; γ is bulk density of solution, (kg / m^3); $k(x, y, z)$ is filtration coefficient, (m/day); t is time. $h(x, y, z)$ is thickness of the ore-bearing horizon (m); β is coefficient of elastic capacity, (cm^2 / kg); α is a constant, taking values 0 or 1, (1/m); $\alpha_1, \alpha_2, \alpha_3$ are constants, taking values 0 or 1; ξ is coefficient for reducing equations to dimensions (1/day);

$$f(x, y, z, t) = \sum_1^n q_i(t) \delta(x - x_i, y - y_i, z - z_i); \quad q_i(t) \quad \text{is well flow rates;}$$

$$\delta = \begin{cases} 1, & x = x_i, y = y_i, z = z_i \\ 0, & \text{other condation} \end{cases} \quad \text{is Dirac delta function.}$$

Conclusion

This study has provided a comprehensive mathematical model for analyzing the hydrodynamic processes involved in ISL within a three-dimensional porous medium. By incorporating filtration, diffusion, and kinetic factors, the model enables a deeper

understanding of how pressure variations, permeability, and porosity impact the extraction process. The use of numerical methods for solving complex quasilinear partial differential equations has proven effective in simulating real-world ISL conditions.

A key finding of this research is the significant influence of pressure changes on hydrodynamic parameters, particularly in response to solution injection and extraction. These variations directly affect the efficiency of mineral recovery and the potential environmental consequences, such as groundwater contamination. The proposed mathematical framework allows for improved monitoring, forecasting, and optimization of ISL operations, ensuring both economic feasibility and environmental safety.

Future research should focus on refining this model by incorporating more dynamic factors such as chemical interactions, multiphase flow behavior, and real-time field data integration. Further advancements in computational algorithms will also enhance the precision and applicability of ISL simulations, leading to more sustainable and efficient mining practices.

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