APPLICATION OF THE FINITE DIFFERENCE METHOD IN THE DISCRETIZATION AND SOLUTION OF THE EULER EQUATIONS

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Abstract: *A comparative study of finite difference schemes for the Euler partial differential equations is conducted. In addition, a stability analysis of the difference schemes is performed. Two-step finite difference methods are employed to solve the Euler equations.*

Keywords: *Euler equations, Lax scheme, MacCormack scheme, explicit two-step scheme, finite difference scheme.*

Introduction: This article presents and provides a detailed study of various finite difference schemes that can be used to solve simple model equations. The analysis is limited to the Euler equations. The Euler equations are fundamental equations of hydrodynamics that describe the motion of an ideal fluid flow and account for the forces acting on the fluid. In the Euler model, the fluid is considered ideal, meaning that thermal conductivity is neglected (the fluid maintains a constant temperature, with no heating or cooling), and viscosity is absent (no frictional forces arise within the fluid). Therefore, the forces acting on such a fluid are reduced to pressure forces, as well as gravitational and inertial forces[1].

1. Lax Scheme

$$\frac{u_j^{n+1}-u_j^n}{\Delta t}+c\frac{u_{j+1}^n-u_{j-1}^n}{2\Delta x}=0 \qquad (1)$$

$$\frac{u_j^{n+1}-(u_{j+1}^n+u_{j-1}^n)/2}{\Delta t}+c\frac{u_{j+1}^n-u_{j-1}^n}{2\Delta x}=0 \qquad (2)$$

2. MacCormack Scheme

Predictor
$$U_j^{\overline{n+1}}=U_j^n-c\frac{\Delta t}{\Delta x}(U_{j+1}^n-U_j^n) \qquad (3)$$

Corrector
$$U_j^{\overline{n+1}}=\frac{1}{2}\left[U_j^n+U_j^{\overline{n+1}}-c\frac{\Delta t}{\Delta x}(U_j^{\overline{n+1}}+U_{j-1}^{\overline{n+1}})\right] \qquad (4)$$

Mathematical Model. Euler Equations [2].

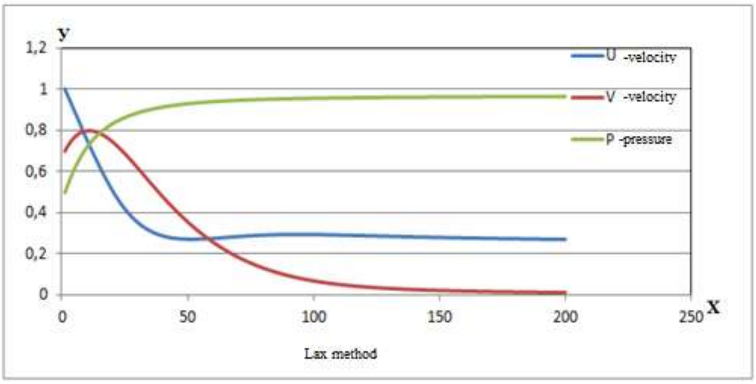
$$\begin{cases} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} = 0 \\ \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial y} = 0 \\ \frac{\partial P}{\partial t} + k(\frac{\partial u}{\partial x} + \frac{\partial V}{\partial y}) = 0 \\ \frac{\partial u}{\partial x} + \frac{\partial V}{\partial y} = 0 \end{cases} \quad (5)$$

The following formula is used for the numerical solution of the equation:[3].,

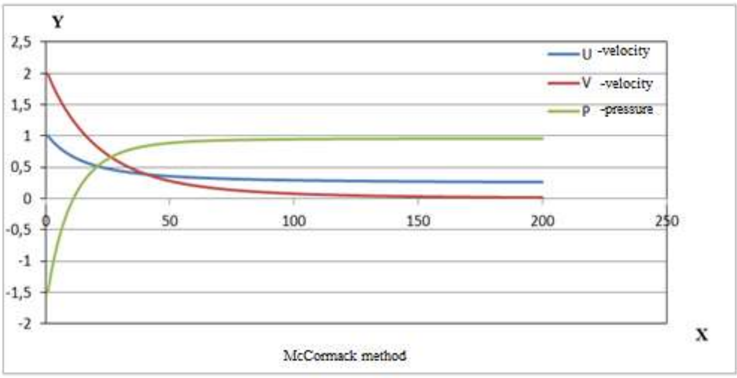
$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial x} = \frac{\partial \xi}{\partial y}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta} \quad (6)$$

Results of Calculations.

Figure 1 presents a comparison of the results obtained using the Lax method and the MacCormack method for velocity and pressure.



a)



b)

Conclusion: A comparison of the calculation results has been carried out. It is shown that these finite difference schemes are stable. These schemes can be effectively used for solving more complex problems in hydrodynamics.

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