

APPLICATION OF THE LEONTIEF MODEL IN SOLVING ECONOMIC PROBLEMS

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Abstract. *This paper considers the application of Wassily Leontief's linear input–output model to the analysis and solution of practical economic problems, in particular the construction of inter-sectoral balance equations. The historical roots of the model, its mathematical formulation in terms of the matrix of technical coefficients, and the conditions under which the corresponding system of balance equations possesses a meaningful economic solution are discussed. A numerical example illustrating the computation of the total output vector for a three-sector economy is presented, together with a brief overview of the theoretical and applied directions in which the model continues to be developed.*

Keywords: *balance equations, sector, matrix of technical coefficients, input–output analysis, Leontief inverse, productive matrix.*

Introduction. The problem of describing the interdependence of many branches of material production arises naturally whenever the output of one sector of the economy is required, in one form or another, as an input for another sector. The systematic mathematical treatment of this problem — commonly referred to today as input–output analysis — goes back to the work of the American economist of Russian origin Wassily Leontief, who in the 1930s constructed a numerical “Tableau Économique” of the United States economy and, on this basis, proposed a system of linear balance equations linking the gross output of every sector to the amounts it delivers to all other sectors and to final consumption [3].

Although the model is purely quantitative in its original formulation, its conceptual origin can be traced to the reproduction schemes of the Physiocrats and, later, to the classical political economy of Ricardo and Marx; a detailed account of this logical–historical development, connecting Leontief's matrices to Sraffa's system of production prices and to Oskar Lange's theory of economic planning, is given in [1]. Regardless of the theoretical tradition in which it is placed, the practical value of the model lies in the fact that it reduces a large and seemingly disorganized set of inter-sectoral transactions to a single matrix equation that can be solved, once and for all, for the vector of total outputs corresponding to any given vector of final demand.

Mathematical Formulation of the Model

Suppose the material production of a country (or a region) is divided into n interdependent sectors $S_1, S_2, \dots, S_n$. Let x_i denote the total output of sector i over a fixed period, and let q_{ij} denote the quantity of the product of sector i that is consumed by sector

j in the process of producing x_j . The remaining part of the output of sector i , which is not spent on production and is instead directed to consumption, export, or accumulation, is called the sector's final (or net) product and is denoted y_i . For every sector the following balance identity must hold:

$$x_i = q_{i1} + q_{i2} + \dots + q_{in} + y_i, \quad i = 1, \dots, n.$$

Dividing each term q_{ij} by the output x_j of the consuming sector produces the so-called technical (technological) coefficient $a_{ij} = q_{ij} / x_j$, which shows how many units of the product of sector i are required, at the existing level of technology, to produce one unit of the product of sector j . Substituting $q_{ij} = a_{ij} x_j$ into the balance identity yields the system

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n + y_i, \quad i = 1, \dots, n,$$

which, written in matrix form with X and Y denoting the column vectors of total and final output and $A = [a_{ij}]$ the $n \times n$ matrix of technical coefficients, becomes

$$X = AX + Y, \quad \text{i.e. } (E - A)X = Y,$$

where E is the identity matrix of order n . This is precisely the linear equation that Leontief proposed in 1936 as a mathematical model of a multi-sector economy [4]; the matrix A is accordingly called the matrix of direct (or bevosita) input requirements, and equation $(E - A)X = Y$ is known as Leontief's linear economic model.

Provided the matrix $(E - A)$ is non-singular, the total-output vector corresponding to any prescribed vector of final demand is obtained as

$$X = (E - A)^{-1} Y = P \cdot Y, \quad P = (E - A)^{-1},$$

where P is called the matrix of full (indirect and direct) requirements, or Leontief's inverse; each entry p_{ij} of P shows the total amount of output of sector i , direct and indirect, that is ultimately needed to deliver one unit of final product of sector j to the economy.

Existence and Economic Meaningfulness of the Solution

For the model to be of practical use, the system $(E - A)X = Y$ must possess a solution X that is not only mathematically well defined but also economically meaningful, that is, a solution with non-negative (in the productive case, strictly positive) components. This requirement is closely related to the classical properties of non-negative matrices: the Hawkins–Simon condition, according to which every leading principal minor of $(E - A)$ must be positive, is equivalent to the matrix A being “productive”, i.e., to the existence of some positive output vector X for which $(E - A)X$ is likewise positive.

A rigorous and fairly complete treatment of when the balance system $(E - A)x = d$ admits a non-negative and economically meaningful solution — including the more delicate case in which the technical-coefficients matrix fails to be irreducible, so that the economy decomposes into several strongly connected blocks of sectors — is carried out in [2], where the existence and (up to normalisation) uniqueness of the solution is expressed in terms of the spectral radius of the diagonal blocks of A and of the associated directed graph of inter-sectoral dependencies. These results also underlie the sensitivity analysis of the model, that is, the study of how strongly the computed sectoral outputs react to small errors or deliberate changes in the estimated technical coefficients.

A Numerical Illustration

Consider a simplified economy consisting of three sectors — agriculture (S1), industry (S2), and services (S3) — for which the matrix of technical coefficients and the vector of final demand (in conventional monetary units) are given by

<i>0.20</i>	<i>0.30</i>	<i>0.10</i>	<i>Y1 = 180</i>
<i>0.15</i>	<i>0.25</i>	<i>0.20</i>	<i>Y2 = 220</i>
<i>0.10</i>	<i>0.10</i>	<i>0.15</i>	<i>Y3 = 140</i>

All leading principal minors of $(E - A)$ are here positive, so by the Hawkins–Simon condition the matrix A is productive and the system $(E - A)X = Y$ has a unique, economically meaningful solution. Solving the corresponding system of three linear equations gives, approximately,

$$x_1 \approx 341, \quad x_2 \approx 424, \quad x_3 \approx 271$$

as the required total gross outputs of agriculture, industry, and services, respectively. The example shows how a modest amount of statistical information — the transaction table underlying the coefficients a_{ij} and the planned final demand — allows one to determine, by a single matrix inversion, the volume of production that every sector of the economy must reach in order to satisfy both inter-sectoral requirements and the needs of final consumption.

Concluding Remarks

The Leontief model remains one of the most transparent and, at the same time, most powerful instruments of applied economic analysis: it reduces the whole of the technological structure of an economy to a single square matrix and allows one to trace, through the Leontief inverse, the full chain of direct and indirect consequences of a change in demand for any single product. Contemporary research continues to develop the model along several directions — the rigorous study of existence and uniqueness of solutions for reducible technology matrices [2], the reconstruction of its connections to classical and Marxist political economy through the work of Sraffa and Lange [1], and its extension to dynamic, regional, multi-regional, and environmentally oriented input–output models [3]. For teaching purposes, however, the value of the model lies chiefly in the fact that it gives students a concrete, computable illustration of how a system of linear algebraic equations can serve as a faithful mathematical image of the material interdependence of an entire economy [4].

References

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