

MUSBAT ANIQLANGAN MATRITSALAR VA ULARNING XOSSALARI.

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Annotatsiya. Ushbu maqolada Musbat aniqlangan matritsasi haqida tushuncha , Musbat aniqlangan matritsasini qo'llash , Musbat aniqlangan matritsasiga oid bir nechta teoremlar va misollar berildi. Musbat aniqlangan matritsasi matematikada Algebra fanida ko'p uchraydi.

Annotation. Annotation. This paper provides an overview of the Positive matrix, the application of the Positive matrix, and several theorems and examples related to the Positive matrix. The Positive matrix is common in mathematics and in the science of algebra and functional analysis.

Kalit so'zlar: musbat matritsa, argument, kompleks, vektor, matritsa, transponerlash, musbat matritsa, funksiya, simetrik

Key words: Positive, argument, complex, vector, matrix, transponder, positive matrix, function, symmetric

Bizga C^n fazodan n –o'lchamli H gilbert fazosi berilgan bo'lsin. Ikki x va y vektorlar vektor ko'paytmasini $\langle x, y \rangle$ yoki $x * y$ ko'rinishida ifodalaymiz. Barcha chiziqli operatorlar fazosi H ni $L(H)$ orqali belgilaymiz va $n \times n$ kompleks matritsalarini $M_n(C)$ yoki M_n fazo orqali belgilaymiz. $L(H)$ fazoning har bir elementi A , C^n ning standart bazisi $\{e_j\}$ matritsani aniqlanadi. A orqali biron matritsani belgilaymiz.

1.1.1-ta'rif. Agarda A matritsa uchun quyudagi shart bajarilsa

$$\langle x, Ax \rangle \geq 0 \text{ barcha } x \in H \quad (1.1.1)$$

U holda A matritsa musbat aniqlangan matritsa deyiladi.

1.1.2-ta'rif. Agarda A matritsa uchun quyudagi shart bajarilsa

$$\langle x, Ax \rangle > 0 \text{ barcha } x \neq 0 \quad (1.1.2)$$

U holda A matritsa qat'iy musbat aniqlangan matritsa deyiladi .

A qat'iy musbat aniqlangan matritsa, musbat aniqlangan deyiladi faqat va faqat A ning teskari mavjud bo'lsa.

Qisqacha qilib, qat'iy musbat aniqlangan yoki musbat aniqlangan matritsalarini bir nom bilan musbat matritsa termini ishlatiladi. Bazida, agar musbat aniqlangan matritsaga urg'u qaratilsa, bu qat'iy musbat bo'ladi. A musbat matritsani bildirishi uchun, $A \geq 0$ ni belgilash ishlatiladi va $A > 0$ qat'iy musbatligini bildiradi.

A musbat matritsalarini juda xam ko'p xossalari mavjut. Misol uchun:

(i) A musbat matritsa bo'ldi agarda Hermitian ($A = A^*$) va barcha xos qiymatlari nomanfiy bo'lsa. A qat'iy musbat matritsa deyiladi agarda uning barcha xos qiymatlari musbat bo'lsa.

(ii) A musbat matritsa bo'ldi agarda Hermitian va barcha bosh minorlari nomanfiy bo'lsa. A qat'iy musbat matritsa deyiladi agarda uning barcha bosh minorlari musbat bo'lsa

(iii) A musbat matritsa bo'ldi agarda ba'zi B matritsalar uchun $A = BB^*$, A qat'iy musbat matritsa deyiladi agarda B yagona bo'lmasa.

(iv) A musbat matritsa bo'ldi agarda $A = TT^*$ ba'zin T yuqori uchburchakli matritsa uchun

(v) A musbat matritsa bo'ldi agarda $A = B^2$ ba'zi B musbat matritsalar uchun . Bunda B yagona. Biz $B = A^{\frac{1}{2}}$ deb yozib va A ning kvadrat ildizi

(vi) deyamiz. A qat'iy musbat matritsa deyiladi agarda B qat'iy musbat matritsa bo'lsa.

A musbat matritsa bo'ldi agarda H ning x_1, x_2, \dots, x_n lari mavjud bo'lsa,

$$a_{ij} = \langle x_i, x_j \rangle. \quad (1.1.3)$$

A qat'iy musbat matritsa deyiladi agarda $x_j; 1 \leq j \leq n$ vektorlar chiziqli erkli bo'lsa. Musbat matritsalarini tavsiflovchi bir qator tasdiqlar mavjud. Qulaylik uchun quyida biz ulardan ba'zilarini isbotsiz keltiramiz.

1.1.1-tasdiq. A musbat matritsa bo'lishi uchun u ermit matritsa bo'lib, barcha xos qiymatlari nomanfiy bo'lishi zarur va yetarlidir. A qat'iy musbat bo'lishi uchun esa barcha xos qiymatlari musbat bo'lishi zarur va yetarlidir.

1.1.1-misol Bizga $A = \begin{pmatrix} 3 & i \\ -i & 2 \end{pmatrix}$ Ermit matritsasi berilgan bo'lsin. A matritsani musbatlikka tekshiring.

Demak A matritsamiz Ermit, xos qiymatlarini topsak, ya'ni $A - \lambda I = 0$

$$|A - \lambda I| = \begin{vmatrix} 3 - \lambda & i \\ -i & 2 - \lambda \end{vmatrix} = \lambda^2 - 5\lambda + 5 = 0; \quad \lambda_{1,2} = \frac{5 \pm \sqrt{5}}{2}$$

$\lambda_{1,2} > 0$. Bundan, A matritsa qat'iy musbat.

1.1.2-tasdiq. A musbat matritsa bo'lishi uchun u ermit matritsa bo'lib, barcha bosh minorlarining determinanti nomanfiy bo'lishi zarur va yetarlidir. A qat'iy musbat bo'lishi uchun esa barcha bosh minorlarining determinanti musbat bo'lishi zarur va yetarlidir.

1.1.2-misol Bizga $A = \begin{pmatrix} 1 & 2-i & i \\ 2+i & 2 & 5 \\ -i & 5 & 3 \end{pmatrix}$ Ermit matritsasi berilgan bo'lsin. A matritsani musbatlikka tekshiring.

A matritsani bosh minorlarining determinantini topamiz

$$|A_1| = |1| = 1; \quad |A_2| = \begin{vmatrix} 1 & 2-i \\ 2+i & 2 \end{vmatrix} = -3$$

$$|A_3| = |A| = \begin{vmatrix} 1 & 2-i & i \\ 2+i & 2 & 5 \\ -i & 5 & 3 \end{vmatrix} = -46$$

Demak A matritsaning barcha bosh minorlari musbat emasligidan, A matritsa musbat aniqlanmagan.

1.1.3-tasdiq. A musbat matritsa bo'lishi uchun shunday B matritsa topilib, $A = B^* \cdot B$ bo'lishi zarur va yetarlidir. A qat'iy musbat bo'lishi uchun esa B singular bo'lmagan matritsa bo'lishi zarur va yetarlidir.

1.1.3-misol Bizga $A = \begin{pmatrix} 9 & 5-i & i+1 \\ 5-i & 6 & 4+i \\ i+1 & 4+i & 4 \end{pmatrix}$ matritsa berilgan bo'lsin. A matritsani

musbatlikka tekshiring

$$B = \begin{pmatrix} 3 & 1 & -i \\ 1 & 2 & 1 \\ i & 1 & 2 \end{pmatrix} \text{ matritsani tanlab olib unga qo'shma } B^* \text{ quyudagicha}$$

$$B^* = \begin{pmatrix} 3 & 1 & i \\ 1 & 2 & 1 \\ -i & 1 & 2 \end{pmatrix} \text{ bo'ladi. } A = B^* \cdot B \text{ shartga tekshirib ko'ramiz}$$

$$B^* \cdot B = \begin{pmatrix} 3 & 1 & -i \\ 1 & 2 & 1 \\ i & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 & i \\ 1 & 2 & 1 \\ -i & 1 & 2 \end{pmatrix} = \begin{pmatrix} 9 & 5-i & i+1 \\ 5-i & 6 & 4+i \\ i+1 & 4+i & 4 \end{pmatrix} = A$$

Haqiqatdan o'rinli. Demak, A matritsa musbat aniqlangan matritsa.

1.1.4-tasdiq. A musbat matritsa bo'lishi uchun shunday B musbat matritsa topilib, $A = B^2$ tenglik bajarilishi zarur va yetarlidir. A matritsa qat'iy musbat bo'lishi uchun B ning qat'iy musbat bo'lishi zarur va yetarlidir.

4-tasdiqdagi B matritsa yagona bo'lib, unga A matritsaning kvadratik ildizi deyiladi va $B = A^{1/2}$ kabi belgilanadi.

1.1.4- misol Bizga $A = \begin{pmatrix} 5 & 9 & 7i \\ 4 & 10 & 5i \\ 4i & 6i & -1 \end{pmatrix}$ matritsa berilgan bo'lsin. A matritsani

musbatlikka tekshiring

$$B = \begin{pmatrix} 2 & 2 & i \\ 1 & 3 & i \\ i & i & 1 \end{pmatrix} \text{ matritsani tanlab olamiz va } A = B^2 \text{ shart bajarilishini tekshiramiz.}$$

$$B^2 = \begin{pmatrix} 2 & 2 & i \\ 1 & 3 & i \\ i & i & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & i \\ 1 & 3 & i \\ i & i & 1 \end{pmatrix} = \begin{pmatrix} 5 & 9 & 7i \\ 4 & 10 & 5i \\ 4i & 6i & -1 \end{pmatrix} = A$$

Haqiqatdan o'rinli. Demak, A matritsa musbat aniqlangan matritsa.

Navbatdagi tasdiqni bayon qilish uchun L orqali Evklid fazosini, ya'ni skalyar ko'paytma kiritilgan chiziqli fazoni belgilaymiz.

1.1.5-tasdiq. $A = (a_{ij}) \in M_n(C)$ musbat bo'lishi uchun shunday $z_1, z_2, \dots, z_n \in L$ elementlar topilib,

$$a_{ij} = (z_i, z_j), 1 \leq i, j \leq n$$

tengliliklar bajarilishi zarur va yetarlidir. A qat'iy musbat bo'lishi uchun z_j , $1 \leq j \leq n$ elementlar chiziqli bog'lanmagan bo'lishi zarur va yetarlidir.

1.1.5- misol: A matritsa quyudagicha berilgan:

$$A = \begin{pmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{pmatrix}$$

Matritsani musbat aniqlanganlikda tekshiring.

$$A - \lambda I = 0$$

$$\begin{vmatrix} 25 - \lambda & 15 & -5 \\ 15 & 18 - \lambda & 0 \\ -5 & 0 & 11 - \lambda \end{vmatrix} = 0$$

$$(25 - \lambda) \cdot (18 - \lambda) \cdot (11 - \lambda) - 25(18 - \lambda) - 225(11 - \lambda) = 0$$

$$-\lambda^3 + 54\lambda^2 - 673\lambda + 2025 = 0$$

$$\lambda^3 - 54\lambda^2 + 673\lambda - 2025 = 0$$

Idizlarni Nyuton Rafson usuli yordamida topish mumkin

Nyuton Rafson usuli

$$\lambda^3 - 54\lambda^2 + 673\lambda - 2025 = 0$$

$f(x) = x^3 - 54x^2 + 673x - 2025$ bu funksiyaning hosilasini topamiz

$$f'(x) = 3x^2 - 108x + 673$$

$x_0 = 4$ nuqatada funksiya va uning hosilasining qiymatini topamiz.

$$f(x_0) = f(4) = 4^3 - 54 \cdot 4^2 + 673 \cdot 4 - 2025 = -133$$

$$f'(x_0) = f'(4) = 3 \cdot 4^2 - 108 \cdot 4 + 673 = 289$$

x_1 nuqta quyudagicha aniqlangan

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 4 - \frac{-133}{289}$$

$$x_1 = 4.46020761$$

$x_1 = 4.46020761$ nuqatada funksiya va uning hosilasining qiymatini topamiz

$$f(x_1) = f(4.46020761)$$

$$= 4.46020761^3 - 54 \cdot 4.46020761^2 + 673 \cdot 4.46020761 - 2025$$

$$= -8.7977561$$

$$f'(x_1) = f'(4.46020761) = 3 \cdot 4.46020761^2 - 108 \cdot 4.46020761 + 673$$

$$= 250.97793369$$

x_2 nuqtani x_1 kabi quyudagicha aniqlaymiz

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4.46020761 - \frac{-8.7977561}{250.97793369} = 4.49526152$$

x_2 nuqatada funksiya va uning hosilasining qiymatini topamiz

$$f(x_2) = f(4.49526152)$$

$$= 4.49526152^3 - 54 \cdot 4.49526152^2 + 673 \cdot 4.49526152 - 2025$$

$$= -0.04986905$$

$$f'(x_2) = f'(4.49526152) = 3 \cdot 4.49526152^2 - 108 \cdot 4.49526152 + 673 \\ = 248.13388462$$

x_3 nuqtani aniqlaymiz

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 4.49526152 - \frac{-0.04986905}{248.13388462} = 4.49546249$$

x_3 nuqatada funksiya va uning hosilasining qiymatini topamiz

$$f(x_3) = f(4.49546249) \\ = 4.49546249^3 - 54 \cdot 4.49546249^2 + 673 \cdot 4.49546249 - 2025 \\ = -0.00000164$$

$$f'(x_3) = f'(4.49546249) = 3 \cdot 4.49546249^2 - 108 \cdot 4.49546249 + 673 \\ = 248.11759994$$

x_4 nuqtani aniqlaymiz

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 4.49546249 - \frac{-0.00000164}{248.11759994} = 4.4954625$$

x_4 nuqatada funksiya va uning hosilasining qiymatini topamiz

$x^3 - 54x^2 + 673x - 2025$ tenglamaning taqribiy bir ildizi Nyuton Rafson usulidan $x_1 = 4.4954625$ kelib chiqadi.

N	x_{n-1}	$f(x_{n-1})$	$f'(x_{n-1})$	x_n
1	4	-133	289	4.46020761
2	4.46020761	-8.7977561	250.97793369	4.49526152
3	4.49526152	-0.00000164	248.13388462	4.49546249
4	4.49546249	-0.00000164	248.11759994	4.4954625

Bizga ma'lumki $x^3 - 54x^2 + 673x - 2025 = (x - 4.4954625)(x^2 + ax + b)$

Bundan $x^2 + ax + b$ ni topsak

$$\frac{x^3 - 54x^2 + 673x - 2025}{x - 4.4954625} = x^2 - 49.5045375x + 450.45420817$$

$x^2 - 49.5045375x + 450.45420817 = 0$ tenglamani yechsak

$x_2 = 12.01568365$ va $x_3 = 37.48885386$ bo'ladi.

A matritsaning xos qiymatlari λ 4.4954625; 12.01568365 ; 37.48885386 lar musbat u holda A matritsa ham musbat.

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