CLASSIFICATION FOUR DIMENSIONAL ω LIE ALGEBRA IN A CASE (RANK 2).

Mirzobek Shodiev
Bukhara State University
mirzobekshodiyev7@gmail.com

Abstract. *I give the algebraic classification of complex four-dimensional ω-Lie algebras.*

Keywords: *ω-Lie algebra, Jordan canonic form, bilinear, Jacobi identity*

In this case, we choose $\{e_1, e_2\}$ as a basis of L' and $e_3, e_4 \notin L'$. We assume that $[e_1, e_2] = ae_1 + by$ and $[e_3, e_4] = me_1 + ne_2$. Our arguments will be separated into the following two cases: $a = bm = n = 0$ or the others.

If all a, b, m, n are zero, then $[e_1, e_3], [e_1, e_4], [e_2, e_3], [e_2, e_4], \neq 0$ because the kernel of φ is two dimensional. Therefo the linear map ad $_{e_3}: L' \to L'$ by $u \mapsto [e_3, u]$ and ad $_{e_4}: L' \to L'$ by $u \mapsto [e_4, u]$ is bijective. By course of linear algebra, we can appropriately select basis elements e_1 and e_2 so that ad $_{e_3}$ is similar to

$$\mathbb{M}_{11}: \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} or \quad \mathbb{M}_{12}: \begin{pmatrix} \lambda & 0 \\ 1 & \lambda \end{pmatrix},$$

and ad $_{e_4}$ is similar to

$$\mathbb{M} : \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Here, $c,\ d,\ \lambda,\ a_{11},\ a_{12},\ a_{21},\ a_{22}$ are all non-zero complex numbers.

for case $\mathbb{M}_{11}$ as follows: $[e_3, e_1] = ad_{e_3}(e_1) = ce_1$ and $[e_3, e_2] = ad_{e_3}(e_2) = de_2$.

for case $\mathbb{M}_{12}$ as follows : $[e_3, e_1] = ad_{e_3}(e_1) = \lambda e_1 + e_2$ and $[e_3, e_2] = ad_{e_3}(e_2) = \lambda e_2$.

for case $\mathbb{M}$ as follows : $[e_4, e_1] = ad_{e_4}(e_1) = a_{11}e_1 + a_{21}e_2$ and $[e_4, e_2] = ad_{e_4}(e_2) = a_{12}e_1 + a_{22}e_2$.

subcase 1 In the first situation, ad $_{e_3}$ similar to $\mathbb{M}_{11}$ and ad $_{e_4}$:

$[e_1, e_2] = 0$; $[e_3, e_1] = ce_1$; $[e_3, e_2] = de_2$; $[e_4, e_1] = a_{11}e_1 + a_{21}e_2$; $[e_4, e_2] = a_{12}e_1 + a_{22}e_2$; $[e_3, e_4] = 0$;

By ω-Jacobi identity, we have

$i)$ $\omega(e_1, e_2)e_3 + \omega(e_2, e_3)e_1 + \omega(e_3, e_1)e_2 = [[e_1, e_2], e_3] + [[e_2, e_3], e_1] + $
$+[[e_3, e_1], e_2] = 0$

$$ii)\ \omega(e_1,e_2)e_4+\omega(e_2,e_4)e_1+\omega(e_4,e_1)e_2=0$$

$$iii)\ \omega(e_1,e_3)e_4+\omega(e_3,e_4)e_1+\omega(e_4,e_1)e_3=[[e_1,e_3],e_4]+ +[[e_3,e_4],e_1]+[[e_4,e_1],e_3]=0$$

we determine this equality : $\omega(e_i,e_j)=0.\ i,j\in(1,4)\ \ and i\neq j$

This indicates that all ω are zero, this means we take trivial ω-Lie algebra

subase 2 In the first situation, ad $_{e_3}$ similar to $\mathbb{M}_{12}$ and ad $_{e_4}$:

$[e_1,e_2]=0$ $[e_3,e_1]=\lambda e_1$ $[e_3,e_2]=e_1+\lambda e_2$ $[e_4,e_1]=a_{11}e_1+a_{21}e_2$ $[e_4,e_2]=a_{12}e_1+a_{22}e_2$ $[e_3,e_4]=0$

By ω-Jacobi identity, we have

This indicates that all ω are zero, this means we take trivial ω -Lie algebra

Case 2 Suppose one of a and b is non-zero. We only need to focus on the scenario where $b\neq 0$, as if $a\neq 0$, we can interchange e_1 and e_2 and obtain equivalent outcomes.

Let $\widetilde{e_1}=e_1+a^{-1}be_2$, then $[\widetilde{e_1},e_2]=[e_1,e_2]=ae_1+be_2=be_1$. Let $\widetilde{e_2}=b^{-1}e_2$ then $[\widetilde{e_1},\widetilde{e_2}]=\widetilde{e_1}$. So in this case, we can assume that $[e_1,e_2]=e_1$. Since the kernel of φ is two dimensional, two of $[e_3,e_1]$, $[e_3,e_2]$, $[e_4,e_1]$, $[e_4,e_2]$ and $[e_3,e_4]$ is zero, and the other is not zero.

We are fixed $[e_3,e_1]=0$, another situation izomorf each other.

$[e_1,e_2]=e_1$, $[e_3,e_1]=0$, $[e_3,e_2]=0$, $[e_4,e_1]=ae_1+by$, $[e_4,e_2]=ce_1+de_2$, $[e_3,e_4]=me_1+ne_2$,

By ω-Jacobi identity, we have

Thus, $\omega(e_2,e_3)=\omega(e_4,e_2)\omega(e_3,e_4)=o$, This indicates that all ω are zero, this means we take trivial ω -Lie algebra

Lemma 2 When classifying any n-dimensional ω-Lie algebras, it is sufficient to consider the cases when the rank of φ (i.e., the dimension of L′) is n − 1 and n.

REFERENCES

1. W. Fulton and J. Harris, Representation theory: a first course. GTM 129. Springer-Verlag, New York (1991).

2. P. Nurowski, Deforming a Lie algebra by means of a 2-form. J. Geom. Phys. 57 (2007) 1325-1329.