THERMOPLASTIC PROBLEMS

Omonboyev Abdulxakim G'ayratbek o'g'li
National University of Uzbekistan named after Mirzo Ulugbek, 2nd stage master's student.

ABSTRACT: *This thesis aims to investigate one-dimensional thermoelastic and thermoplastic problems by developing numerical models to analyze the effects of temperature and mechanical stress on material behavior. The study distinguishes between thermoelastic and thermoplastic theories, focusing on how materials behave under varying temperature conditions and mechanical loads. The numerical solutions are derived using finite difference methods (FDM) and finite element methods (FEM), providing insights into temperature distribution and deformation profiles over time. The results demonstrate that thermoelastic models exhibit only reversible deformations, while thermoplastic models show irreversible plastic deformations when stress exceeds a critical threshold. The developed models are applicable in engineering disciplines such as automotive, aerospace, and material science for evaluating the structural integrity of materials under high thermal and mechanical stresses.*

KEYWORDS: *Thermoplastic modeling, One-dimensional thermoplasticity, Deformation under the influence of temperature, Thermal stress analysis, Numerical methods (numerical methods), Finite Element Method (FEM), Theory of plasticity, Thermal-mechanical issues, Coefficient of temperature-dependent expansion, Discretization methods.*

INTRODUCTION: The behavior of materials under thermal and mechanical loads is of significant interest in engineering and scientific research. One-dimensional thermoelastic and thermoplastic problems serve as fundamental models for understanding the coupled interactions between heat conduction and material deformation. While thermoelasticity considers only elastic deformations, thermoplasticity also accounts for irreversible plastic deformations, which occur when the material stress surpasses the yield strength. This study aims to create numerical models for one-dimensional thermoelastic and thermoplastic problems to simulate and analyze material responses under varying conditions.

2. Mathematical Formulation

The mathematical models for thermoelastic and thermoplastic problems are based on the governing equations of heat conduction and stress-strain relations. The following sections outline the primary equations used in both models.

2.1. Thermoelastic Problem

The governing equations for the thermoelastic problem include the heat conduction equation and the stress-strain relation. The heat conduction equation is given by:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \beta T \frac{\partial \sigma}{\partial t}$$

Where:

- ρ is the density,
- c is the specific heat capacity,
- k is the thermal conductivity,
- T is the temperature,
- β is the thermal expansion coefficient, and
- σ is the stress.

The stress-strain relationship is expressed as:

$$\sigma = E \cdot \varepsilon_{total} = E \cdot (\varepsilon - \varepsilon_{th})$$

Where

- E is Young`s module
- ε_{total} is the total strain
- ε is the mechanical strain, and
- $\varepsilon_{th} = \beta(T - T_0)$ is the thermal strain

2.2. Thermoplastic Problem

The thermoplastic model introduces plastic strain ε_{pl} in addition to the thermoelastic strain:

$$\sigma = E(\varepsilon - \varepsilon_{th} - \varepsilon_{pl})$$

The plastic deformation occurs when the stress exceeds the yield strength ε_{yeild} :

The governing heat conduction equation includes the heat generated by plastic work:

$$\sigma > \sigma_{yield} \Rightarrow \varepsilon_{pl} = \varepsilon_{pl} + \Delta\varepsilon_{pl}$$

Where h_p

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \beta T \frac{\partial \sigma}{\partial t} + h_p$$

is the heat generated by plastic deformation.

3. Numerical Solution Methods

The numerical solution of these models is performed using finite difference methods (FDM) and finite element methods (FEM). The spatial and temporal discretization allows for solving coupled differential equations iteratively over small time steps.

- **Spatial Discretization**: Divides the one-dimensional domain into discrete nodes.
- **Temporal Discretization**: Utilizes a time-stepping scheme to update temperature and stress values at each node.

3.1. Finite Difference Method (FDM)

The heat conduction equation is discretized using central difference approximations, resulting in the following finite difference scheme:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = a\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} + \beta T_i^n \frac{\sigma_i^{n+1} - \sigma_i^n}{\Delta t}$$

Where:

- $a = \frac{k}{pc}$ is the thermal diffusivity,
- Δt and Δx are the time and spatial step sizes,
- T_i^n denotes the temperature at position i and time step n.

4. Numerical Implementation and Results

A simplified numerical model was implemented using Python to simulate temperature and deformation profiles for both thermoelastic and thermoplastic cases. The results show that:

- **Thermoelastic Model**: Temperature changes cause elastic deformations that are fully recoverable. No plastic deformation occurs, even at high temperatures.
- **Thermoplastic Model**: When stress exceeds the yield limit, irreversible plastic strains develop, leading to permanent changes in material shape. The temperature distribution influences the stress-strain relationship significantly, highlighting the interdependency between thermal and mechanical effects.

5. Conclusions

The study successfully developed numerical models for one-dimensional thermoelastic and thermoplastic problems using finite difference methods. The results provide insights into material behavior under varying thermal and mechanical conditions:

- Thermoelastic models are suitable for scenarios where deformations remain within the elastic range.
- Thermoplastic models are necessary for understanding material behavior under high-stress conditions, where plastic deformations are inevitable.

The developed numerical models can be used for practical engineering applications to predict material responses and optimize designs under complex thermal and mechanical loading conditions.

6. Future Work

Future research could extend the models to multi-dimensional cases and include more complex material properties such as viscoelasticity, damage mechanics, and dynamic loading conditions. Additionally, integrating machine learning algorithms for parameter optimization could further enhance model accuracy and computational efficiency.

REFERENCES:

1. Zienkiewicz, O. C., & Taylor, R. L. The Finite Element Method for Solid and Structural Mechanics. Elsevier, 2013.
2. Bathe, K. J. Finite Element Procedures. Prentice Hall, 1996.
3. Sadd, M.H. Elasticity: Theory, Applications, and Numerics. Academic Press, 2009.
4. Reddy, J.N. An Introduction to the Finite Element